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A new Closure operator-GRW-closure in Topological Spaces

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Abstract

In this paper, GRW-open sets , GRW-neighbourhoods,GRW-interior and GRW-closure are introduced and some of their basic properties are studied .GRW-int(A) and GRW-Cl(A)
(ie cl^{GRW}(A)) are defined and prove that it forms a topology τ_{GRW} in X.

Key words and phrases: GRW-int(A), GRW-cl(A)(cl^{GRW}(A)) , τ_{GRW} GRW-neighborhood is abbreviated as GRW-nbd.

1 Introduction

N. Levine[11] introduced generalized closed sets in general topology as a generalization of closed sets. This concept was found to be useful and many results in general topology were improved. Many researchers like Balachandran, Sundaram and Maki[3], Bhattacharyya and Lahiri[4], Arockiarani[1], Dunham[9], Gnanambal[10], Malghan[15], Palaniappan and Rao[19], Park[20], Arya and Gupta[2] and Devi[8] have worked on generalized closed sets, their generalizations and related concepts in general topology.Pushpalatha and Rajarubi[21] introduced GRW-closed sets in a topological spaces .

In this section, GRW-open sets in topological spaces and obtain some of their properties. Also, GRW-neighbourhood in topological spaces by using the notion of GRW-open sets .

Moreover in this paper, the notion of GRW-interior is defined and some of its basic properties are studied.Also the concept of GRW-closure in topological spaces using the notions of GRW-closed sets, and some related results will be obtained . The τ_{GRW} is defined and prove that it forms a topology on X. For any A ⊂ X, it is proved that the complement of GRW-interior of A is the GRW-closure of the complement of A.

Throughout the paper, X and Y denote the topological spaces (X, τ) and (Y, σ) respectively and on which no separation axioms are assumed unless Otherwise explicitly stated. For any subset A of a space (X, τ), the closure of A, interior of A, w-interior of A, w-closure of A, gpr-interior of A, gpr-closure of A, α-closure of A, α-interior of A and the complement of A are denoted by cl (A) or τ -cl (A), int (A) or τ -int (A), w-int (A), w-cl (A), gpr-int (A), gprcl (A), α-int (A), α-cl (A) and A^c or X − A respectively. Sometimes (X, τ) is denoted by simply X if there is no confusion arise.

2 Preliminaries

The following definitions are used as preliminary .

Definition 2.1. A subset A of a space X is called

1) a **preopen set** [16] if A ⊆ intcl (A) and a **preclosed set** if clint (A) ⊆ A.

2) a α**-open set** [18] if A ⊆ intclint (A) and a α**-closed set** if clintcl (A) ⊆ A.

3) a **regular open set** [23] if A = intcl (A) and a **regular closed set** if A = clint (A).

The intersection of all preclosed (resp. α-closed) subsets of X containing A is called pre-closure (resp. α-closure) of A and is denoted by pcl(A)(resp. α-cl (A)).

Definition 2.2. A subset A of a space X is called

1) **generalized** α**-closed set** (briefly, gα-closed) [13] if αcl (A) ⊆ U whenever A ⊆ U and U is α-open in X.

2) α**-generalized closed set** (briefly, αg-closed) [14] if αcl (A) ⊆ U whenever A ⊆ U and U is open in X.

3) **regular generalized closed set** (briefly, rg-closed) [19] if cl (A) ⊆ U whenever A ⊆ U and U is regular open in X.

4) **generalized preclosed set** (briefly, gp-closed) [12] if pcl (A) ⊆ U whenever A ⊆ U and U is open in X.

5) **weakely generalized closed set** (briefly, wg-closed) [17] if cl int (A) ⊆ U whenever A ⊆ U and U is open in X.

6) **weakely closed set** (briefly, w-closed) [22] if cl (A) ⊆ U whenever A ⊆ U and U is semi open in X.

7)**g-rg-closed set(rg*-closed)[21]** if cl*(A) ⊆ U whenever A ⊆U and U is regular open in X.

8)**rw-closed set**[6]] if cl (A) ⊂ U whenever A ⊂ U and U is regular semi open in X.

The complements of the above mentioned closed sets are their respective open sets.

Definition 2.3. A subset A of a space X is called **regular semi open set** (briefly, RS-open) [4] if there is a regular open set U such that U ⊂ A ⊂cl(U).

Definition 2.4. A subset A of a space X is called a **generalized regular weakly closed set** (briefly,GRW-closed) [21] if cl* (A) ⊂ U whenever A ⊂ U and U is regular semi open in X.

The set of all GRW-closed sets in X by GRWC(X).

Remark 2.1[21] Every w-closed set is GRW-closed .

Remark 2.2[21] Every GRW-closed set is g-rg-closed.

Remark 2.3[21] Complement of Regular semi open set is Regular semi open.

Remark2.4[21]w-int(A)= $\cup\{G : G\, is\, a\, w-open\, set, G\subseteq A\}$

Remark 2.5[21] If A and B are GRW-closed sets in a topological space then A ∪ B is also GRW-closed set.

3 .GRW-open sets and GRW-neighbourhoods.

In this section, GRW-open sets in topological spaces and obtain some of their properties. Also, GRW-neighbourhood in topological spaces by using the notion of GRW-open sets. It will be proved that every neighbourhood of x in X is GRW-neighbourhood of x but not conversely.

Definition 3.1. A subset A in X is called generalized rw-open (briefly, GRW-open) in X if A^C is GRW-closed in X.The family of all GRW-open sets in X by GRWO (X).

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Theorem 3.1. If a subset A of a space X is w-open then it is GRW-open but not conversely.

Proof. Let A be a w-open set in a space X. Then A^c is w-closed set. By Remark 2.1 A^c is GRW-closed. Therefore A is GRW-open set in X.

The converse of the above theorem need not be true, as seen from the following example.

Example 3.1. 8 Let X = {a, b, c, d} be with the topology

τ = {X, ϕ, {b}, {c}, {a, b},{b,c}.{a, b, c}}.

GRW-open-{ {a},{b},{c},{d},{a,c}, {a, d}, {b,c},{b,d},{a,b},{a,b,c} }
GRW--closed sets { X, ϕ, {d}, {a,c}, {a, d}, {b,c},{b,d},{c,d}, {a, b, c, }, {a, b, d},{a,c,d},{b,c,d}}
W-open sets {{a,b,c} ,{b,c},{a,b},{c},{b}, X, ϕ} {a} is GRW-open but not w-open and not open in X.

Corollary 3.1. Every open set is GRW-open set but not conversely.

Corollary 3.2. Every regular open set is GRW-open set but not conversely.

Proof. Every regular open set is open so its complement is closed, closed sets are GRW-closed. Hence every regular open set is GRW-open.

Theorem 3.2. If a subset A of a space X is GRW-open, then it is g-rg-open set in X.

Proof. Let A be GRW-open set in space X. Then A^c is GRW-closed set in X.

By Remark 2.2 , A^c is GRW-closed then it is g-rg-closed set in X. Therefore A is g-rg-open set in space X.

The converse of the above theorem need not be true, as seen from the following example.

Example 3.2. Let X = {a, b, c, d,e} with topology

τ = {X, φ, {a} , {d} , {e} , {a, d} , {a, e} , {d, e} , {a, d, e}}. In this topological space the subset {a, b} is g-rg-open but not GRW-open set in X.

Theorem 3.4. If a subset A of a topological space X is RW-open, then it is GRW-open set in X, but not conversely.

Proof. Let A be rw-open set in a space X. Then A^c is rw-closed set in X.

rw-closed set A^c is GRW-closed in X. Therefore A is GRW-open subset in X.

The converse of the above theorem need not be true, as seen from the following example.

Example 3.3. Let X = {a, b, c, d} with topology, τ = {X, ϕ, {b}, {c}, {a, b},{b,c}.{a, b, c}}.

{a,c} is GRW-open set in X, but not rw-open set .

Theorem 3.5. If A and B are GRW-open sets in a space X. Then A ∩ B is also GRW-open set in X.

Proof. If A and B are GRW-open sets in a space X. Then A^c and B^c are GRW-closed sets in a space X. By $A^c \cup B^c$ is also GRW-closed set in X. That is $A^c \cup B^c = (A \cap B)^c$ is a GRW-closed set in X. Therefore A ∩ B GRW-open set in X.

Example 3.4. The union of two GRW-open sets in X need not be a GRW-open set in X.

Let X = {a, b, c, d} with topology, τ = {X, ϕ, {b}, {c}, {a, b},{b,c}.{a, b, c}}.{c,d} is not GRW-open set in X, but {c} and {,d} are GRW-open sets in X,{c} $\bigcup$ {d}={c,d}

Theorem 3.6. If a set A is GRW-open in a space X, such that G is regular semi open and int (A) ∪ A^c ⊂ G , then G = X.

Proof. Suppose that A is GRW-open in X. Let G be regular semi open set such that

int (A) ∪ A^c ⊂ G , A^c ⊂ int (A) ∪A^c⊂ G ,A^c is GRW-closed hence Cl*(A^c) ⊂ G. This implies G^c ⊂ Cl*(A^c) - A^c ,but G^c is RSO and A^c is GRW-closed ((A) ∪ A^c)^c = (int (A))^c ∩ A. That is

G^c ⊂ (int (A))^c – A^c, Thus G^c ⊂ cl* (A)^c – A^c, Since (int (A))^c = cl (A^c)=. Now G^c is also regular semi open and A^c is GRW-closed, by Remark 2.3, it follows that G^c = φ. Hence G = X.

The converse of the above theorem is not true in general as seen from the following example.

Example 3.5. Let X = {a, b, c, d} with topology

τ = {X, φ, {a} , {b} , {a, b} , {b, c} , {a, b, c} , {a, b, d}}. Then RSO (X) =

{X, φ, {a} , {b} , {c} , {d} , {a, b} , {b, c} , {c, d} , {a, d} , {b, d} , {a, c} , {a, b, c} ,

{a, b, d}} and RO (X) = {X, φ, {a} , {b} , {c} , {b, c} , {a, d} , {b, c, d}}. Take

A = {b, c, d}. Then A is not GRW-open. However int (A) ∪A^c = {b, c}∪{a} =

{a, b, c}. So for some regular semi open G, int (A) ∪ A^c = {a, b, c} ⊂ G gives G = X, but A is not GRW-open.

Theorem 3.7. Every singleton set in a topological space is GRW-open.

Proof. Let X be a topological space. Let x ∈ X. To prove {x} is GRW-open, it is enough to prove that X − {x} is GRW-closed.

Let U be a RSO and X − {x}⊂ U, but X is the only RSO such that X − {x}⊂ U, so U=X therefore Cl*(X − {x})⊂X hence X- {x} is

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GRW-closed that is {x} is GRW-open.

GRW-neighborhoods in a topological Space.

Definition 3.2. Let X be a topological space and let $x \in X$. A subset N of X is said to be a GRW- neighborhood of x if and only if there exists a GRW-open set G such that $x \in G \subset N$.

Definition 3.3. A subset N of space X, is called a GRW- neighborhood of $A \subset X$ if and only if there exists a GRW-open set G such that $A \subset G \subset N$.

Remark 3.1 The GRW- neighborhood N of $x \in X$ need not be a GRW-open in X.

Example 3.6 Let X = {a, b, c, d} be with the topology τ = {X, ϕ, {b}, {c}, {a, b},{b,c}. {a, b, c}}.

Set of all GRW-closed sets in (X, τ) is

{ X, ϕ, {d}, {a,c}, {a, d}, {b,c},{b,d},{c,d},{a, b, c, }, {a, b, d},{a,c,d},{b,c,d} }

Set of all GRW-open sets is

GRWO (X) = { X, ϕ, {a},{b},{c},{d},{a,b},{a,c},{a, d}, {b,c},{b,d},{a,b,c} }

The set {c,d} is GRW- neighborhoods of the points c and d since the GRW-open sets {c}and {d} such that $c \in \{c\} \subset \{c,d\}$ and $d \in \{d\} \subset \{c,d\}$ .However, the set {c,d} is not a GRW-open set in X .

Theorem 3.8. Every neighborhood N of $x \in X$ is a GRW- neighborhood of X.

Proof. Let N be a neighborhood of point $x \in X$. To prove that N is a GRW- neighborhood of x. By the definition of GRW-neighborhood, there exists an open set G such that $x \in G \subset N$. As every open set is GRW-open set G such that $x \in G \subset N$. Hence N is GRW- neighborhood of x.

Remark 3.2. In general, a GRW- neighborhood N of $x \in X$ need not be a neighborhood of x in X, as seen from the following example.

Example 3.7 . Let X = {a, b, c, d}with topology τ = {X, φ, {a} , {b} , {a, b} , {a, b, c}}.

GRW--closed sets in (X, τ) { X, ϕ, {d}, {a, b}, {c, d}, {a,d},{b,d},{a, b, c, }, {a, b, d},{a, c, d}, {b, c, d} }

GRWO(X)=GRW-open sets in X = { X, ϕ, {a},{b},{c},{d},{a, b}, {a,c},{b, c}, {c, d},{a,b,c} }, {c,d} is a GRW- neighborhood of c since {c} is a GRW-open set such that $c \in \{c\} \subset \{c,d\}$, but it is not a neighborhood of c.

Theorem 3.9. If a subset N of a space X is GRW-open, then N is a GRW- neighborhood of each of its points.

Proof. Suppose N is GRW-open. Let $x \in N$. It will be proved that N is GRW- neighborhood of x. For N is a GRW-open set such that $x \in N \subset N$. Since x is an arbitrary point of N, it follows that N is a GRW- neighborhood of each of its points.

Remark 3.3. The converse of the above theorem is not true in general as seen from the following example.

Example 3.8. Let X = {a, b, c, d} be with the topology

τ = {X, ϕ, {b}, {c}, {a, b},{b,c}. {a, b, c}}.

Set of all GRW-closed sets in (X, τ) is

{ X, ϕ, {d}, {a,c}, {a, d}, {b,c},{b,d},{c,d},{a, b, c, }, {a, b, d},{a,c,d},{b,c,d}}

Set of all GRW-open sets is

GRWO (X) = { X, ϕ, {a},{b},{c},{d},{a,b},{a,c},{a, d}, {b,c},{b,d},{a,b,c} }

The set {c,d} is GRW- neighborhood of the points c and d since the GRW-open sets {c}and {d} such that $c \in \{c\} \subset \{c,d\}$ and $d \in \{d\} \subset \{c,d\}$ .However, the set {c,d} is not a GRW-open set in X .

Theorem 3.10. Let X be a topological space. If F is a GRW-closed subset of X, and $x \in F^c$. Prove that there exists a GRW- neighbourhood N of x such that $N \cap F = \varphi$.

Proof. Let F be GRW-closed subset of X and $x \in F^c$. Then F^c is GRW-open set of X. So by theorem 3.9. F^c contains a GRW- neighbourhood of each of its points. Hence there exists a GRw- neighbourhood N of x such that $N \subset F^c$. That is $N \cap F = \varphi$.

Definition 3.4. Let x be a point in a space X. The set of all GRW- neighbourhood of x is called the GRW- neighbourhood system at x, and is denoted by GRW-N (x).

Theorem 3.11. Let X be a topological space and for each $x \in X$, Let GRW-N (x) be the collection of all GRW- neighbourhood of x. Then the followings are true .

(i) $\forall x \in X$, GRW-N (x) $\neq \varphi$.

(ii) $N \in$ GRW-N (x) $\Rightarrow x \in N$.

(iii) $N \in$ GRW-N (x), $M \supset N \Rightarrow M \in$ GRW-N (x).

(iv) N ∈ GRW-N (x), M ∈ GRW-N (x) ⇒ N ∩M ∈ GRW-N (x).

(v) N ∈ GRW-N (x) ⇒ there exists M ∈ GRW-N (x) such that M ⊂ N and

M ∈ GRW-N (y) for every y ∈ M.

Proof. (i) Since X is a GRW-open set, it is a GRW- neighbourhood of every x ∈ X. Hence

there exists at least one GRW- neighbourhood (namely - X) for each x ∈ X.

Hence GRW-N (x) ≠ φ for every x ∈ X.

(ii) If N ∈ GRW-N (x), then N is a GRW- neighbourhood of x. So by definition of GRW- neighbourhood ,x ∈ N.

(iii) Let N ∈ GRW-N (x) and M ⊃ N. Then there is a GRW-open set G such

that x ∈ G ⊂ N. Since N ⊂ M, x ∈ G ⊂ M and so M is GRW- neighbourhood of x.

Hence M ∈ GRW-N (x).

(iv) Let N ∈ GRW-N (x) and M ∈ GRW-N (x). Then by definition of GRW- neighbourhood

there exists GRW-open sets G1 and G2 such that x ∈ G1 ⊂ N and x ∈ G2 ⊂ M.

Hence x ∈ G1 ∩ G2 ⊂ N ∩M → (1). Since G1 ∩ G2 is a GRW-open set, (being

the intersection of two GRW-open sets), it follows from (1) that N ∩ M is a

GRW- neighbourhood of x. Hence N ∩M ∈ GRW-N (x).

(v) If N ∈ GRW-N (x), then there exists a GRW-open set M such that x ∈ M ⊂

N. Since M is a GRW-open set, it is GRW- neighbourhood of each of its points. Therefore

M ∈ GRW-N (y) for every y ∈ M.

Theorem 3.12. Let X be a nonempty set, and for each x ∈ X, let GRW-N (x)

be a nonempty collection of subsets of X satisfying following conditions.

(i) N ∈ GRW-N (x) ⇒ x ∈ N

(ii) N ∈ GRW-N (x),M ∈ GRW-N (x) ⇒ N ∩M ∈ GRW-N (x).

Let τ consists of the empty set and all those non-empty subsets of G of X having the property that x ∈ G implies that there exists an N ∈ GRW-N (x)

such that x ∈ N ⊂ G, Then τ is a topology for X.

Proof. (i) φ ∈ τ by definition. It will be proved that X∈ τ . Let x be any arbitrary element of X. Since GRW-N (x) is nonempty, there is an N ∈ GRW-

N (x) and so x ∈ N by (i). Since N is a subset of X, x ∈ N ⊂ X. Hence X ∈ τ .

(ii) Let G1 ∈ τ and G2 ∈ τ. If x ∈ G1 ∩ G2 then x ∈ G1 and x ∈ G2. Since

G1 ∈ τ and G2 ∈ τ , there exists N ∈ GRW-N (x) and M ∈ GRW-N (x), such

that x ∈ N ⊂ G1 and x ∈ M ⊂ G2. Then x ∈ N ∩ M ⊂ G1 ∩ G2. But N ∩M ∈ GRW-N (x) by (2). Hence G1 ∩ G2 ∈ τ .

(iii) Let G_λ ∈ τ for every λ ∈ ∧ . If x ∈ ∪{ G_λ : λ ∈ ∧ },

then x ∈ $G_{\lambda x}$ for some λx ∈ ∧ . Since $G_{\lambda x}$ ∈ τ , there exists an N ∈ GRW-N (x) such that x ∈ N ⊂ $G_{\lambda x}$ and consequently x ∈ N ⊂ ∪{ G_λ : λ ∈ ∧ }.

Hence ∪{ G_λ : λ ∈ ∧ } ∈ τ. It follows that τ is topology for X.

4. GRW-interior and GRW-closure

Definition 4.1. Let A be a subset of X. A point x ∈ A is said to be **GRW-interior point of** A if A is a GRW-nbd of x. The set of all GRW-interior points of A is called the GRW-interior of A and is denoted by GRW-int(A).

Theorem 4.1. If A be a subset of X. Then GRW-int(A) = ∪{G : G is GRW-open, G ⊆ A}.

Proof. Let A be a subset of X.

x ∈ GRW-int (A) ⇔x is a GRW-interior point of A.

⇔A is a GRW-nbd of point x.

⇔there exists GRW-open set G such that x ∈ G ⊆ A.

⇔x ∈ ∪{G : G is GRW-open, G ⊆ A}.

Hence GRW-int (A) = ∪{G : G is GRW-open, G ⊆ A}.

Theorem 4.2. Let A and B be subsets of X. Then

(i) GRW-int(X) = X and GRW-int(ϕ) = ϕ .

(ii) GRW-int(A) ⊆ A.

(iii) If B is any GRW-open set contained in A, then B ⊆ GRW-int(A).

(iv) If A ⊆ B, then GRW-int(A) ⊆ GRW-int(B).

(v) GRW-int(GRW-int(A)) = GRW-int(A).

Proof. (i) Since X and ϕ are GRW-open sets, by Theorem 2.1. GRW-int (X) =

∪{G : G is GRw-open, G ⊆ X} = X ∪ {all GRW-open sets } = X. That isGRW-

int (X) = X. Since ϕ is the only GRW-open set contained in ϕ , GRW-int (ϕ) = ϕ .

(ii) Let x ∈ GRW-int (A) ⇒ x is a GRW-interior point of A.

⇒ A is a GRW-nbd of x.

⇒ x ∈ A.

Thus x ∈ GRW-int (A) ⇒ x ∈ A. Hence GRW-int (A) ⊆ A.

(iii) Let B be any GRW-open sets such that B ⊆ A. Let x ∈ B, then since

B is a GRW-open set contained in A. x is a GRW-interior point of A. That is

x ∈ GRW-int (A). Hence B ⊆ GRW-int (A).

(iv) Let A and B be subsets of X such that A ⊆ B. Let x ∈ GRW-int (A). Then x is a GRW-interior point of A and so A is GRW-nbd of x. Since B ⊃ A, B is also a GRW-nbd of x. This implies that x ∈ GRW-int (B). Thus we have shown that x ∈ GRW-int (A) ⇒ x ∈ GRW-int (B). Hence GRW-int (A) ⊂ GRW-int (B).

(v) From (ii) and (iv) GRW-int(GRW-int(A)) ⊆ GRW-int(A).

Let x ∈ GRW-int(A) this imply A is a neighborhood of x ,so there exist a GRW-open set G such that x ∈ G ⊆ A so every element of G is an GRW-interior of A ,hence x ∈ G ⊆ GRW-int(A) which means that x is an GRW-interior point of GRW-int(A) that is GRW-int(A) ⊆ GRW-int(GRW-int(A)).That is GRW-int(GRW-int(A)) = GRW-int(A).

Let A be any subset of X. By the definition of GRW-interior GRW-int(A) ⊂ A by (iii) GRW-int(GRW-int(A)) ⊂GRW-int(A). Hence GRW-int(GRW-int(A)) ⊂ ∩ {F : A ⊂ F ∈ GRWC(X)} = GRW-cl(A).

Theorem 4.3. If a subset A of space X is GRW-open, then GRW-int (A) = A.

Proof. Let A be GRW-open subset of X. GRW-int (A) ⊂ A. Also, A is GRW-open set contained in A. From Theorem 4.2. (iii) A ⊂ GRW-int (A).

Hence GRW-int (A) = A.

The converse of the above Theorem need not be true, as seen from the following example.

Example 4.1 Let X = {a, b, c, d} with topology τ = {X, φ, {a} , {b} , {a, b} , {a, b, c}}.

GRW-closed sets is

{ X, ϕ, {d}, {a, b}, {c, d}, {a,d},{b,d},{a, b, c, }, {a, b, d},{a, c, d}, {b, c, d}} .

GRWO(X) is GRW-open sets in X = { X, ϕ, {a},{b},{c},{d},{a, b}, {a,c},{b,c},{c, d},{a,b,c} }

GRW-int (A) =GRW-int ({a,d}}={a} ∪ {d} ={a,d} ,but {a,d} is not GRW-open set.

Theorem 4.4. If A and B are subsets of X, then GRW-int (A)∪GRW-int (B) ⊂ GRW-int (A ∪ B).

Proof. For ant sub sets A and B of X , A ⊂ A ∪ B and B ⊂ A ∪ B. By Theorem 4.2. (iv), GRW-int (A) ⊂ GRW-int (A ∪ B) and GRW-int (B) ⊂ GRW-int (A ∪ B). This implies that GRW-int (A) ∪ GRW-int (B) ⊂ GRW-int (A ∪ B).

Theorem 4.5. If A and B are subsets of space X, then GRW-int (A ∩ B) = GRW-int (A) ∩ GRW-int (B).

Proof. For ant sub sets A and B of X ,A ∩ B ⊂ A and A ∩ B ⊂ B. By Theorem 2.2. (iv), GRW-int (A ∩ B) ⊂ GRW-int (A) and GRW-int (A ∩ B) ⊂ GRW-int (B).

This implies that GRW-int (A ∩ B) ⊂ GRW-int (A) ∩ GRW-int (B) → (1).

Again, let x ∈ GRW-int (A) ∩ GRW-int (B). Then x ∈ GRW-int (A) and x ∈ GRW-int (B).

Hence x is a GRW-interior point of each of sets A and B. It follows that A and B are GRW-nbds of x, so that their intersection A∩B is also a GRW-nbds of x.

Hence x ∈ GRW-int (A ∩ B). Thus x ∈ GRW-int (A) ∩ GRW-int (B) implies that x ∈ GRW-int (A ∩ B).

Therefore , GRW-int (A) ∩ GRW-int (B) ⊂ GRW-int (A ∩ B)→(2).

From (1) and (2),

GRW-int (A ∩ B) = GRW-int (A)∩GRW-int (B).

Theorem 4.6. If A is a subset of X, then int (A) ⊂ GRW-int (A).

Proof. Let A be a subset of a space X.

Let x ∈ int(A) ⇒ x ∈ ∪{G : G is open, G ⊂ A}.

This imply that, there exists an open set G such that x ∈ G ⊂ A.

This imply that ,there exist a GRW-open set G such that x ∈ G ⊂ A, as every open set is a GRW-open set in X.

This imply that ,x ∈ ∪{G : G is GRW-open, G ⊂ A}.

This imply that x ∈ GRW-int (A).

Thus x ∈ int (A), this imply that x ∈ GRW-int (A). Hence int (A) ⊂ GRW-int (A).

Remark 4.1. Converse of the above theorem need not be true.

Example 4.2 Let X = {a, b, c} with topology τ = {X, φ, {a},{b}, {a, b}}.

Then GRWO(X) = The power set of X . Let A = {a,c}. GRW-int (A) = {a, c} and int (A) = {a}. It follows that int (A) ⊂ GRW-int (A) and GRW-int (A) ⊄ int (A).

Theorem 4.7. If A is a subset of X, then w-int(A) ⊂ GRW-int(A), where w-int(A) is given by w-int(A) = ∪{G : G is a w-open, G ⊂ A}by remark 2.4

Proof. Let A be a subset of a space X.

Let x ∈ w-int(A)

(ie) x ∈ ∪{G ⊂ X : G is a w-open, G ⊂ A}.

(ie) there exists a w-open set G such that x ∈ G ⊂ A.

(ie) there exists a GRW- open set G such that, x ∈ G ⊂ A,

as every w-open set is a GRW-open set in X.

(ie) x ∈ ∪{G ⊂ X : G is a GRW-open, G ⊂ A}.

(ie) x ∈ GRW-int (A).

Thus x ∈ w-int (A) implies x ∈ GRW-int (A). Hence w-int (A) ⊂ GRW-int (A).

Remark 4.2. Containment relation in the above Theorem4.7 may be proper as seen from the following example.

Example 4.3 Let X = {a, b, c} with topology τ = {X, φ, {a}}, Then GRWO(X) = P(X) and WO(X) = {X, φ, {a}}. Let A = {a, b}. Then GRW-int(A) = {a, b} and w-int (A) = {a}. It follows that w-int (A) ⊂ GRW-int (A) and GRW-int (A) ⊄ w-int(A).

Theorem 4.8. If A is a subset of X, then GRW-int (A) ⊂ g-rg-int (A), where g-rg-int (A) is given by g-rg-int (A) = ∪{G ⊂ X : G is a g-rg-open, G ⊂ A}.

Proof. Let A be a subset of a space X.

Let x ∈ GRW-int (A) = ∪{G ⊂ X : G is a GRW-open, G ⊂ A}.

(i e) there exists a GRW-open set G such that x ∈ G ⊂ A.

(i e) there exists a g-rg-open set G such that $x \in G \subset A$,
as every GRW-open set is g-rg-open set in X.
(i e) $x \in \cup\{G \subset X : G$ is a g-rg-open, $G \subset A\}$.
(i e) $x \in$ g-rg-int (A).
Thus $x \in$ GRW-int (A) implies that $x \in$ g-rg-int (A).
Hence GRW-int (A) $\subset$ g-rg-int (A).

GRW-closure in a space X .

Definition 4.2. Let A be a subset of a space X. The GRW-closure of A is defined as the intersection of all GRW-closed sets containing A.
GRW-cl (A) = $\cap\{F : A \subset F \in$ GRWC (X)$\}$.
Theorem 4.9. If A and B are subsets of a space X .Then
(i) GRW-cl (X) = X and GRW-cl (ϕ) = ϕ. (ii)$A \subset$ GRW-cl (A).
(iii) If B is any GRW-closed set containing A, then GRW-cl (A) $\subset$ B.
(iv) If $A \subset B$ then GRW-cl (A) $\subset$ GRW-cl (B).
(v) GRW-cl(A) = GRW-cl(GRW-cl(A)).
Proof. (i) By the definition of GRW-closure, X is the only GRW-closed set containing X. Therefore GRW-cl (X) = Intersection of all the GRW-closed sets containing X. = $\cap\{X\}$ = X. That is GRW-cl (X) = X. By the definition of GRW-closure, GRW-cl (ϕ) = Intersection of all the GRW-closed sets containing φ = ∩any GRW-closed sets containing ϕ = ϕ. That is GRW-cl (ϕ) = ϕ.
(ii) By the definition of GRW-closure of A , it is obvious that $A \subset$ GRW-cl (A).
(iii) Let B be any GRW-closed set containing A. Since GRW-cl (A) is the intersection of all GRW-closed sets containing A, GRW-cl (A) is contained in every GRW-closed set containing A. Hence in particular GRW-cl (A) $\subset$ B.
(iv) Let A and B be subsets of X such that $A \subset B$. By the definition of GRW-closure, GRW-cl (B) = $\cap\{F : B \subset F \in$ GRWC (X)$\}$. If $B \subset F \in$ GRWC (X), then GRW-cl (B) $\subset$ F. Since $A \subset B$, $A \subset B \subset F \in$ GRWC (X), GRW-cl (A) $\subset$ F. Therefore GRW-cl (A) $\subset \cap\{F : B \subset F \in$ GRWC (X)$\}$ = GRW-cl (B). That is GRW-cl (A) $\subset$ GRW-cl (B).
(v) Let A be any subset of X. By the definition of GRW-closure, GRW-cl (A) = $\cap\{F : A \subset F \in$ GRWC (X)$\}$, If $A \subset F \in$ GRWC (X), then GRW-cl (A) $\subset$ F.
Since F is GRW-closed set containing GRW-cl (A), by (iii) GRW-cl(GRW-cl(A)) $\subset$ F.
Hence GRW-cl(GRW-cl(A)) $\subset \cap\{F : A \subset F \in$ GRWC (X)$\}$ = GRW-cl (A). That is GRW-cl(GRW-cl(A)) = GRW-cl (A).
Theorem 4.10. If $A \subset X$ is GRW-closed, then GRW-cl (A) = A.
Proof. Let A be GRW-closed subset of X. By the definition of GRW-cl(A) ,$A \subset$ GRW-cl (A). Also $A \subset A$ and A is GRW-closed. By Theorem 4.9. (iii) GRW-cl (A) $\subset$ A. Hence GRW-cl (A) = A.
The converse of the above Theorem need not be true as seen from the following example.
Example 4.4
Let X = {a, b, c, d} with topology τ = {X, φ, {a} , {b} , {a, b} , {a, b, c}}.
GRW-closed sets is
{ X, ϕ, {d}, {a, b}, {c, d}, {a,d},{b,d},{a, b, c, }, {a, b, d},{a, c, d}, {b, c, d}},
GRWO(X)=GRW-open sets in X = { X, ϕ, {a},{b},{c},{d},{a, b}, {a,c},{b,c},{c,},{a,b,c}}
GRW-cl (A) =GRW-cl ({a,c})={a,b,c} ∩ {a,c,d} ={a,c} ,but {a,c} is not GRW-closed set.
Theorem 4.11. If A and B are subsets of a space X, then GRW-cl ($A \cap B$) $\subset$ GRW-cl (A) $\cap$ GRW-cl (B).
Proof. Let A and B be subsets of X. Clearly $A \cap B \subset A$ and $A \cap B \subset B$. By Theorem 2.9.(iv), GRW-cl ($A \cap B$) $\subset$ GRW-cl (A) and GRW-cl ($A \cap B$) $\subset$ GRW-cl (B). Hence GRW-cl ($A \cap B$) $\subset$ GRW-cl (A) $\cap$ GRW-cl (B).
Theorem 4.12. If A and B are subsets of a space X, then GRW-cl ($A \cup B$) = GRW-cl (A) $\cup$ GRW-cl (B).
Proof. Let A and B be subsets of X. Clearly $A \subset A \cup B$ and $B \subset A \cup B$. Hence GRW-cl (A) $\cup$ GRW-cl (B) $\subset$ GRW-cl ($A \cup B$)→(1). Now to prove GRW-cl ($A \cup B$) $\subset$ GRW-cl (A) $\cup$ GRW-cl (B). Let $x \in$ GRW-cl ($A \cup B$) and suppose $x \notin$ GRW-cl (A) $\cup$GRW-cl (B). Then there exists GRW-closed sets A_1 and B_1 with $A \subset A_1$, $B \subset B_1$ and $x \notin A_1 \cup B_1$. $A \cup B \subset A_1 \cup B_1$ and $A_1 \cup B_1$ is GRW-closed set by the Remark 2.5 such that $x \notin A1 \cup B1$. Thus $x \notin$GRW-cl($A \cup B$) which is a contradiction to $x \in$ GRW-cl($A \cup B$). Hence GRW-cl($A \cup B$) $\subset$ GRW-cl(A) $\cup$ GRW-cl(B)→(2). From (1) and (2), GRW-cl($A \cup B$) = GRW-cl(A) $\cup$ GRW-cl(B).
Theorem 4.13. For an $x \in X$, $x \in$ GRW-cl(A) if and only if $V \cap A \neq \phi$ for every GRW-open sets V containing x.
Proof. Let $x \in X$ and $x \in$ GRW-cl(A). To prove $V \cap A \neq \phi$ for every GRW-open set V containing x. Proof by contradiction. Suppose there exists a GRW-open set V containing x such that $V \cap A = \phi$. Then $A \subset X-V$ and $X-V$ is GRW-closed. GRW-cl(A) $\subset X - V$. This shows that $x \notin$GRW-cl(A), which is contradiction. Hence $V \cap A \neq \phi$ for every GRW-open set V containing x.
Conversely, let $V \cap A \neq \phi$ for every GRW-open set V containing x. To prove $x \in$ GRW-cl(A).Proof by contradiction. Suppose $x \notin$GRW-cl(A). Then there exists a GRW-closed subset F containing A such that $x \notin F$, which implies that $x \in X - F$ and $X - F$ is GRW-open. Also $(X - F) \cap A \neq \phi$, which is a contradiction. Hence $x \in$ GRW-cl(A).
Theorem 4.14. If A is subset of a space X, then GRW-cl(A) $\subset$ cl(A) and GRW-cl(A)$\subset$ cl*(A)
Proof. Let A be a subset of a space X. By the definition of closure cl(A),GRW-cl(A) and Cl*(A)
GRW-cl(A) $\subset$ cl(A) and GRW-cl(A) $\subset$ cl*(A).
Remark 4.3. Containment relation in the above Theorem 4.14., may be proper as seen from following example.
Example 4.5 Let X = {a, b, c} with topology τ = {X, φ, {a} , {a, b}}.

Then
GRW-cl({a}) = {a} and cl({a}) = X. It follows that GRW-cl({a}) ⊂ cl({a}) and
GRW-cl({a}) _= cl({a}).

Theorem 4.15. If A is subset of a space X, then GRW-cl(A) ⊂ w-cl(A), where
w-cl(A) is given by w-cl(A) = ∩{F ⊂ X : A ⊂ F and F is w-closed set in X}.

Proof. Let A be a subset of X. By definition of w-closure w-cl(A) = ∩{F ⊂
X : A ⊂ F and F is w-closed set in X}. If A ⊂ F and F is w-closed subset of
X, then A ⊂ F ∈ GRWC(X), because every w-closed is GRW-closed subset in
X. That is GRW-cl(A) ⊂ F. Therefore GRW-cl(A) ⊂ ∩{F ⊂ X : A ⊂ F and F
is w-closed} = w-cl(A). Hence GRW-cl(A) ⊂ w-cl(A).

Remark 4.4. Containment relation in the above Theorem 4.15. may be proper
as seen from following example.

Example 4.6 Let X = {a, b, c} with topology τ = {X, φ, {a} , {b} , {a, b}}.
Let A = {a}. Then GRW-cl(A) = {a} and w-cl(A) = {a, c}. That is GRW-cl(A) ⊂ w-cl(A) and w-cl(A) ⊄ GRW-cl(A).

Theorem 4.15: If τ_{GRW} is a collection of subsets of X such that τ_{GRW} = {U ⊂ X : GRW-cl(U^C)} = U^C},then τ_{GRW} is a topology.

Proof :
(i) Obviously ϕ ,X ∈ τ_{GRW}
(ii) If A,B ∈ τ_{GRW} then A ∩ B ∈ τ_{GRW} by the theorem 4.12.
(III)
$$Let\ A_\lambda \in \tau_{GRW}\ for\ some\ \lambda \in \Lambda, then \bigcup(A_\lambda : \lambda \in \Lambda) \in \tau_{GRW} \sin ce$$
$$GRW - Cl(\cup(A_\lambda : \lambda \in \Lambda))^c = (\cup(A_\lambda : \lambda \in \Lambda))^c$$
Using the theorem 4.11.
Hence τ_{GRW} forms a topology on X generated by GRW-closure.

Theorem 4.16. For any topology τ on X, τ ⊂ τ_W ⊂ τ_{GRW}, where τ_w = {U ⊂
X : w-cl(U^C)} = U^C}Remark 2.6

Proof. τ ⊂ τ_W from Remark 2.6. To prove τ_W ⊂ τ_{GRW}. Let U ∈ τ_w
which implies w-cl(U^c) = U^c, it follows that U^c is a w-closed set. Now U^c is
GRW-closed, as every w-closed set is GRW-closed and so GRW-cl(U^c) = U^c. That
is U ∈ τ_{GRW} and so τ_w ⊂ τ_{GRW}. Hence τ ⊂ τ_W ⊂ τ_{GRW}.

Theorem 4.18. Let A be any subset of X. Then
(i) (GRW-int(A))^C = GRW-cl(A^C) (ii) GRW-int(A) = (GRW-cl(A^C))^C
(iii)GRW-cl(A) = (GRW-int(A^C))^C.

Proof. Let x ∈ (GRW-int(A))^C. Then x ∉ GRW-int(A). That is every GRW-
open set U containing x is such that U ⊄ A. That is every GRW-open set U containing x is such that U ∩ A^c ≠ ϕ . By Theorem 4.13., x ∈ GRW-cl(A^c) ,therefore
(GRW-int(A))^c ⊂ GRW-cl(A^c). Conversely, let x ∈ GRW-cl(A^c). Then by Theorem 4.13., every GRW-open set U containing x is such that U ∩A^c ≠ ϕ .
That is every GRW-open set U containing x is such that U ⊄ A. This implies
by Definition of GRW- interior of A, x ∉ GRW-int(A). That is x ∈ (GRW-int(A))^c
and GRW-cl(A^c) ⊂ (GRW-int(A))^c. Thus (GRW-int(A))^c = GRW-cl(A^c).
(ii) Follows by taking complements in (i).
(iii) Follows by replacing A by A^c in (i).

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